NPRE 447 & 521 Interaction of Radiation with Matter II
Homework Assignments

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1 Homework 1

Attention:

1. Both 447 and 521 students should solve the problems without asterisks. In addition, 521 students should also solve the problems marked with asterisks. 447 students will not get extra credit for solving the problems with asterisks.
2. Write down your name and the course number next to your name clearly.
3. Explanation of the score: our brains typically consume about 0.2 Calories per minute. When actively thinking, our brains can kick it up to burning about 1 Calorie per minute. So instead of assigning each question with points, I will assign with Calories. For example, if a problem is given 10 Calories, it means you will need to burn about 10 Calories to solve the problem and the estimated time to solve the problem is about 10 minutes.
4. Because of the breath and depth of the content of the course, it is only possible to cover the essence during the lectures. One must read the relevant chapters in the textbooks to learn the details and gain deeper understandings.

Readings:

Chapter 3, K. S. Krane, *Introductory Nuclear Physics*, 3rd edition, Wiley (1987).

1.1

What are the energies of an electron, a proton, a neutron, and a photon with wavelengths $\lambda = 1$ Å and 1 fm, respectively? (16 Calories)

1.2

What are the kinetic energy, velocity, and wavelength of thermal ($T = 300$ K, room temperature) neutrons? (9 Calories) Why are thermal neutrons useful for materials studies? Give at least three reasons. (3 Calories)

1.3

What is the radius of gyration R_g of a uniformly distributed spherical nucleus with a radius R? Note: $R_g = \langle r^2 \rangle^{\frac{1}{2}} = \left(\int \rho(r) r^2 dV \right)^{\frac{1}{2}}$. (10 Calories)

1.4

What is the potential $V(r)$ of an electron inside and outside a uniformly distributed spherical nucleus with a radius R and charge Ze? (15 Calories)

1.5

In order to describe the high energy electron scattering cross section of a uniformly distributed spherical nucleus with a radius R, compute the form factor $F(k)$ and the structure factor $S(k)$ of the nucleus. (20 Calories)

1.6

1. Compute the binding energy per nucleon $\frac{B}{A}$ for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U. (8 Calories)
2. Compute the neutron separation energy S_n for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U. (8 Calories)
3. Compute the proton separation energy S_p for ^{4}He, ^{7}Li, ^{56}Fe, and ^{238}U. (8 Calories)

1.7 *

Krane: Page 78, Problem 3.18 (20 Calories)

1.8 *

For the 1-dimensional quantum harmonic oscillator

$$H = T + V = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

define the ladder operators as

$$a = \sqrt{\frac{m\omega}{2\hbar}}(x + \frac{ip}{m\omega})$$
$$a^\dagger = \sqrt{\frac{m\omega}{2\hbar}}(x - \frac{ip}{m\omega})$$

1. Compute $[a, a^\dagger]$, $[\hat{N}, a]$, and $[\hat{N}, a^\dagger]$, where $\hat{N} = a^\dagger a$. (3 Calories)
2. For the n^{th} energy eigenstate $|n\rangle$, compute the expectation values of $\langle n|x|n\rangle$, $\langle n|x^2|n\rangle$, $\langle n|p|n\rangle$, $\langle n|p^2|n\rangle$, and the uncertainty relation quantity $\sigma_x\sigma_p$. (10 Calories)
3. For the n^{th} energy eigenstate $|n\rangle$, compute the expectation values of the kinetic energy $\langle n|T|n\rangle$ and the potential energy $\langle n|V|n\rangle$. (5 Calories)
4. If its initial state is
$$|\Psi(t=0)\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$
what is its state at time t? If we perform measurements of the kinetic energy and potential energy of the system at time t, what do we get? Do they depend on time? (10 Calories)

2 Homework 2

Readings:

Chapter 4, K. S. Krane, *Introductory Nuclear Physics*, 3rd edition, Wiley (1987).

2.1

1. What is the binding energy of deuteron? How do you measure it experimentally? How does it compare with the average binding energy per nucleon B/A? (10 Calories)
2. What are the spin and parity of deuteron? Are the spins of the neutron and proton in a deuteron parallel or anti-parallel? Why? (10 Calories)

2.2

1. In order to solve the 3-D time-independent Schrödinger equation for a 3-D isotropic potential $V(r)$, we use separation of variables. Show that the ordinary differential equation of the radial function $R(r) = \frac{u(r)}{r}$ is

$$-\frac{\hbar^2}{2m}\frac{d^2u(r)}{dr^2} + [V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}] = Eu(r)$$

   (10 Calories)
2. Sketch the neutron-proton interaction $V(r)$. (5 Calories)
3. Solve and sketch the bound s-wave ($l = 0$) radial wave function for the deuteron. (20 Calories)
4. What is the probability of finding the neutron or the proton outside the deuteron radius R? (10 Calories)
5. What is the radius of gyration $R_g = \langle r^2 \rangle^{\frac{1}{2}}$ of the deuteron? (10 Calories)

2.3

Krane: Page 113, Problem 4.3 (40 Calories)

2.4 *

The Hamiltonian of a spin 1/2 system (e.g., a neutron, a proton, or an electron) is

$$\hat{H} = \epsilon_1(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) + \epsilon_2(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|)$$

1. What is the matrix representation of $\hat{H}$ in the $|\uparrow\rangle$ and $|\downarrow\rangle$ basis? (5 Calories)
2. Find its eigenvalues and eigenvectors as linear combinations of $|\uparrow\rangle$ and $|\downarrow\rangle$. Note that $|\uparrow\rangle$ and $|\downarrow\rangle$ are not the energy eigenstates of this system. (10 Calories)
3. If the system starts out in state $|\uparrow\rangle$, what is its state at time t? (10 Calories)

2.5 *

What is the probability of finding the ground state electron of a hydrogen atom inside its nucleus? (10 Calories)

2.6 *

1. Produce pseudo-color plots of the spherical harmonics $Y_l^m(\theta,\phi)$ on a sphere for $l \leq 3$ and all the allowed m. The color should be mapped to the values of $Y_l^m(\theta,\phi)$. (10 Calories)

2. Produce constant surface plots of the real, imaginary, and absolute values of the hydrogen atomic orbitals $\psi_{nlm}(r,\theta,\phi)$ for $n \leq 3$ and all the allowed l and m. (10 Calories)

3. Produce pseudo-color plots of the probability density (2D cut along z axis) of the hydrogen atomic orbitals $\psi_{nlm}(r,\theta,\phi)$ for $n \leq 3$ and all the allowed l and m. The color should be mapped to the values of the probability density $|\psi_{nlm}(r,\theta,\phi)|^2$. (10 Calories)

Attach all of your codes.

3 Homework 3

Readings:

Chapter 4, K. S. Krane, *Introductory Nuclear Physics*, 3rd edition, Wiley (1987).

3.1

1. Explain the meaning of the scattering differential cross section $\frac{d\sigma}{d\Omega}$. (10 Calories)
2. Using the neutron-proton interaction potential derived from the bound state of deuteron, what is the total neutron-proton scattering cross section we computed? What is the experimentally measured value of the total neutron-proton cross section? How do you explain the discrepancy? (10 Calories)
3. Why is water important for nuclear engineering? Give at least three reasons. (10 Calories)

3.2

Krane: Page 114, Problem 4.6 (30 Calories)

3.3 *

1. Prove $[L_x, L_y] = i\hbar L_z$ (5 Calories)
2. Prove $[L^2, L_z] = 0$ (5 Calories)
3. Use the ladder operators, prove $L^2|l,m\rangle = \hbar^2 l(l+1)|l,m\rangle$ and $L_z|l,m\rangle = \hbar m|l,m\rangle$ (20 Calories)
4. Prove $L_\pm|l,m\rangle = \hbar\sqrt{(l \mp m)(l \pm m + 1)}|l, m \pm 1\rangle$ (10 Calories)

4 Homework 4

Readings:

Chapter 9, S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014). Lecture notes before the publication of the book can be found at 1 and 2.

4.1

Generally speaking, what are the common types of interaction of neutrons with matter? Explain each type. (10 Calories)

4.2

Derive the Q-equation:

$$Q = E_3 \left(1 + \frac{m_3}{m_4}\right) - E_1 \left(1 - \frac{m_1}{m_4}\right) - \frac{2}{m_4}\sqrt{m_1 m_3 E_1 E_3} \cos\theta$$

(10 Calories)

4.3

Consider a two-particle, say neutron with mass m and a target nucleus with mass Am, elastic scattering process. The energy of the incident neutron is E. The target nucleus is at rest before the collision.

1. Derive the relation between the energy E' and the outgoing angle θ of the scattered neutron in the Laboratory (L) coordination system. (10 Calories)
2. Show that in the Center-of-Mass (CM) coordination system the magnitude of the velocity each particle does not change before and after the collision. Only their directions change. (10 Calories)
3. Derive the relation between the energy E' and the outgoing angle θ_c of the scattered neutron in the Center-of-Mass (L) coordination system. (10 Calories)
4. Derive the relation between θ and θ_c:

$$\cos\theta = \frac{1 + A\cos\theta_c}{\sqrt{A^2 + 1 + 2A\cos\theta_c}}$$

(10 Calories)

4.4 *

Write down the angular momentum operator $\mathbf{L}$ in spherical coordinates. Show that

$$L^2 = -\hbar^2 \left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

and

$$L_z = -i\hbar \frac{\partial}{\partial \phi}$$

Therefore, the eigen equations of them are indeed the angular equations obtained from the separation of variables. (20 Calories)

4.5 *

If the spin angular momentum of an electron ($\frac{1}{2}\hbar$ along the z-axis) comes from the rotation of a classical sphere of radius r_c, calculate the linear speed of the equator in m/s. The r_c is the so-called classical electron radius, which is obtained by assuming the Coulomb energy is the same as the mass energy: $\frac{1}{4\pi\epsilon_0}\frac{e^2}{r_c} = mc^2$ (20 Calories)

5 Homework 5

Readings:

Chapter 10, S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014). Lecture notes before the publication of the book can be found at 1 and 2.

5.1

1. Sketch the typical energy distribution of scattered neutrons $F(E \to E')$. Explain its physical significance. (10 Calories)

2. What's the average energy loss of the neutrons. (10 Calories)

5.2

Sketch the energy-dependence of the total neutron scattering cross section $\sigma_s(E)$ for carbon and water. Explain the main features of the curves. (30 Calories)

5.3

What are the neutron transmission coefficients of 1 cm thick light and heavy water respectively? (20 Calories)

5.4

1. Explain Compton scattering. (10 Calories)

2. Derive the Compton shift formula:

$$\Delta\lambda = \lambda' - \lambda = \lambda_c(1 - \cos\theta)$$

   where $\lambda_c = \frac{2\pi\hbar}{m_e c}$ is the Compton wavelength. (20 Calories)

3. Show that the kinetic energy of the recoil electron (Compton electron) is

$$T = \hbar\omega \frac{\alpha(1 - \cos\theta)}{1 + \alpha(1 - \cos\theta)}$$

   where $\alpha = \frac{\hbar\omega}{m_e c^2}$ is the ratio of the energy of the photon and the rest energy of the electron. (10 Calories)

5.5 *

Suppose a neutron (or an electron, or a proton) is in the spin state $A\begin{pmatrix} i \\ 1 \end{pmatrix}$.

1. What is the normalization factor A? (5 Calories)

2. If we measure S_x, what values do we get? What is the probability of each value? (5 Calories)

3. If we measure S_y, what values do we get? What is the probability of each value? (5 Calories)
4. If we measure S_z, what values do we get? What is the probability of each value? (5 Calories)
5. What are the expectation values of $\langle S_x \rangle$, $\langle S_x^2 \rangle$, $\langle S_y \rangle$, $\langle S_y^2 \rangle$, $\langle S_z \rangle$, and $\langle S_z^2 \rangle$? Verify the uncertainty relations. (20 Calories)

5.6 *

Consider a neutron in a uniform magnetic field along the z axis $\mathbf{B} = B_0 \hat{z}$.

1. If the initial state is $|\psi(0)\rangle = \begin{pmatrix} \cos \frac{\alpha}{2} \\ \sin \frac{\alpha}{2} \end{pmatrix}$, where α is a known constant, what is the time evolution of the state $|\psi(t)\rangle$? (20 Calories)
2. What are the expectation values of S_x, S_y, and S_z? Note that $\omega = \gamma B_0$ is the Larmor frequency, where γ is the gyromagnetic ratio. (20 Calories)

5.7 *

Construct the spin matrices S_x, S_y, and S_z for a particle with spin 1 (for example, a deuteron). (30 Calories)

6 Homework 6

Readings:

Chapter 11, S. Yip, *Nuclear Radiation Interactions*, World Scientific (2014). Lecture notes before the publication of the book can be found at 1 and 2.

6.1

1. Sketch the angular distribution of Compton scattering. Explain the main features of the curve. (15 Calories)
2. Sketch the energy distribution of Compton scattering. Explain the main features of the curve. (15 Calories)

6.2

What is the fate of an excited atom produced by the photoelectric effect? (15 Calories)

6.3

In the Z (the atomic number of the materials) vs E (the energy of the photon) diagram, sketch which regions are dominated by which processes. (15 Calories)

6.4

Sketch the energy dependence of the attenuation coefficient of photons. Explain the main features of the curve. (15 Calories)

6.5

Sketch the incident particle energy dependence of the stopping power. Explain the main features of the curve. (15 Calories)

6.6

Sketch the Bragg curve of an α particle in air. Explain the main features of the curve. (15 Calories)